GOVERNMENT OF ANDHRA PRADESH

COMMISSIONERATE OF COLLEGIATE EDUCATION

SOLID GEOMETRY

THE PLANE –The Equation Passing Through The Condition

MATHEMATICS

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THE PLANE

EQUATION OF THE PLANE PASSING THROUGH THE POINTS HAVING CONDITIONS PROBLEMS

Def : A plane is a surface such that if any two points are taken on in it ,the line joining them

lies wholly on the surface

Every equation of the first degree in x,y,z represents a plane i.e $ax + by + cz + d = 0$

1.THEOREM : Equation to the plane (π) passing through the point $A = (x_1, y_1, z_1)$ and perpendicular to line (L) with d.r's (a, b, c) is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

Proof : Let P $\in \pi$ and $P = (x, y, z)$. $A = (x_1, y_1, z_1)$ and d.r's of L are (a, b, c)

Now d.r's of $AP = \left(x - x_1, y - y_{1,} z - z_1\right)$.

$\overleftrightarrow{AP} \in \pi \Leftrightarrow \overleftrightarrow{\text{AP}} \perp \text{L} \Leftrightarrow a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

$\therefore$ equation to π is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

1.Problem : Find the equation of the plane through (4,4,0) and perpendicular to the planes $x + 2y + 2z = 5$, $and\ 3x + 3y + 2z - 8 = 0$

Solution : Let π be the required plane . Let (a, b, c) be d.r's of normal to π .since the plane

passes through (4,4,0).

Equation to the plane π is $a(x-4) + b(y-4) + c(z-0) = 0$

But π is perpendicular to $x + 2y + 2z = 5$, $and\ 3x + 3y + 2z - 8 = 0$

$\therefore a + 2b + 2c = 0, 3a + 3b + 2c = 0$ solving these equations we get ,

2 2 1 2

3 2 3 3

$\Longrightarrow \frac{a}{4-6} = \frac{b}{6-2} = \frac{c}{3-6}$

$\Longrightarrow \frac{a}{-2} = \frac{b}{4} = \frac{c}{-3}$ i.e $a = -2,\ b = 4,\ c = -3$

$\therefore$ Equation to π is $-2(x-4) + 4(y-4) - 3(z-0) = 0$

$\Longrightarrow -2x + 4y - 3z - 8 = 0$

$\Longrightarrow 2x - 4y + 3z + 8 = 0$ is required plane equation

2.Problem : Find the equation of a plane passing through the points

(2, 2, 1) (9, 3, 6) and perpendicular to a plane $2x + 6y + 6z = 9$

Solution: Let a, b, c are direction ratios of the normal line to the plane.

We know that Equation of a plane passing through P ($_1$,y_1,z_1) and whose direction ratios of the normal line are a, b, c is

$$\text{a}\,(x - x_1) + b(y - y_1) + c(z - z_1) = 0.$$

☐Equation passes through the point (2, 2, 1) is

$$(x - 2) + b(y - 2) + c(z - 1) = 0 \text{ ----------(1)}$$

But it passes through the point (9, 3, 6)

$$\therefore \text{ a}\,(9 - 2) + b(3 - 2) + c(6 - 1) = 0$$

$$\text{a}(7) + b(1) + c(5) = 0 \qquad \text{----------(2)}$$

Also, it is perpendicular to the plane $2x + 6y + 6z = 9$.

$$\therefore \text{ a}\,(2) + b(6) + c(6) = 0 \text{ ----------(3)}$$

To solve the Equations (2) and (3) using the method of cross multiplication

1	5	7	1
6	6	2	6

$$\Rightarrow \frac{a}{6-30} = \frac{b}{10-42} = \frac{c}{42-2}$$

$$\Rightarrow \frac{a}{-24} = \frac{b}{-32} = \frac{c}{40}$$

$$\Rightarrow \frac{a}{3} = \frac{b}{4} = \frac{c}{-5}$$

Put the values in (1)

$$3(x-2) + 4(y-2) - 5(z-1) = 0$$

$$\Rightarrow 3x - 4y - 5z - 3 = 0$$

Is required plane equation.

3.Problem : Find the equation of a plane through the point (1, 2, -3) and parallel tothe plane $2x - 3y + z - 5 = 0$.

Solution: we know the equation of the parallel plane

$$to\ ax + by + cz + d = 0 \text{ is } ax + by + cz + k = 0.$$

The given plane is $2x - 3y + z - 5 = 0$

And it's parallel plane is $2x - 3y + z + k = 0$ where k is a constant.

But it passes through (1, 2, -3)

$$\therefore 2(1) - 3(2) + (-3) + k = 0 \Rightarrow k = 7$$

Equation of parallel plane $2x - 3y + z + 7 = 0$.

4.Problem : Find the equation of a plane through the point (4, 0, 1) and parallel tothe plane $4x + 3y - 12z + 6 = 0$.

Solution: we know the equation of the parallel plane

$$to\ ax + by + cz + d = 0 \text{ is } ax + by + cz + k = 0.$$

The given plane is $4x + 3y - 12z + 6 = 0$

And it's parallel plane is $4x + 3y - 12z + k = 0$ where k is a constant.

But it passes through $(4, 0, 1)$

$\square$ $4(4) + 3(0) - 12(1) + k = 0 \Rightarrow \mathrm{k} = -4$

Equation of parallel plane $4x + 3y - 12z - 4 = 0$.

EXERCISE PROBLEMS:

1. Find the equation of a plane passing through the point (-1, 3, 2) and perpendicular to the planes $x + 2y + 2z - 5 = 0$, $3x + 3y + 2z - 8 = 0$

2. Find the equation of a plane passing through the points

(1, -2, 2) (-3, 1, -2) and perpendicular to a plane $x + 2y - 3z = 5$.


THANK YOU